Decision processes in temporal discrimination

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A B S T R A C T

The processing dynamics underlying temporal decisions and the response times they generate have received little attention in the study of interval timing. In contrast, models of other simple forms of decision making have been extensively investigated using response times, leading to a substantial disconnect between temporal and non-temporal decision theories. An overarching decision-theoretic framework that encompasses existing, non-temporal decision models may, however, account both for interval timing itself and for time-based decision-making. We sought evidence for this framework in the temporal discrimination performance of humans tested on the temporal bisection task. In this task, participants retrospectively categorized experienced stimulus durations as *short* or *long* based on their perceived similarity to two, remembered reference durations and were rewarded only for correct categorization of these references. Our analysis of choice proportions and response times suggests that a two-stage, sequential diffusion process, parameterized to maximize earned rewards, can account for salient patterns of bisection performance. The first diffusion stage times intervals by accumulating an endogenously noisy clock signal; the second stage makes decisions about the first-stage temporal representation by accumulating first-stage evidence corrupted by endogenous noise. Reward-maximization requires that the second-stage accumulation rate and starting point be based on the state of the first-stage timer at the end of the stimulus duration, and that estimates of non-decision-related delays should decrease as a function of stimulus duration. Results are in accord with these predictions and thus support an extension of the drift–diffusion model of static decision making to the domain of interval timing and temporal decisions.

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1. Introduction

In its simplest form, time tracking ability can modulate an agent's expectancy of events that exhibit some level of temporal predictability (e.g., rewards that are delivered on average every 5 s). Frequently, humans and non-human animals must not only anticipate events but must also make explicit judgments about perceived time. For instance, by comparing the currently perceived time to remembered time intervals, an individual can distribute its responses differentially during a trial between two options that predict reward at different delays in order to maximize reward (e.g., Balcı, Freestone, & Gallistel, 2009; Kheifets & Gallistel, 2012).

Simple, non-temporal perceptual decisions have long been modeled according to principles of rational decision making. Signal detection theory (SDT, Green & Swets, 1966), for example, offers an account of choice proportions that change as a function of a payoff scheme. Evidence-accumulation models such as the drift–diffusion model (DDM) have extended SDT to give detailed accounts of response time

distributions in such tasks (Ratcliff, 1978, 1981, 1985, 1988, 2002). Traditionally, the stimuli used in these experiments are categorized according to the level of some defining sensory quality, such as intensity (e.g., loudness) or some other feature (e.g., direction of motion). In contrast, equivalent theoretical accounts have not been given for decisions in which stimuli are categorized only according to their duration.

Historically, to account for performance in such scenarios, models of performance in two-choice temporal decision tasks – such as temporal bisection (Church & Deluty, 1977), temporal generalization (Church & Gibbon, 1982) and time-left (Gibbon & Church, 1981) – incorporate some form of "comparator" that bases decisions on differences between duration estimates (see Buhusi & Meck, 2005). These studies, however, have primarily focused on choice proportions, and to a large extent have overlooked response times. Consequently, the dynamics of comparison and the relation of these dynamics to interval timing processes themselves have largely been left unexamined (although see Leon & Shadlen, 2003; Kononowicz & van Rijn, 2011; Ng, Tobin, & Penney, 2011). This disconnect between analytical approaches to temporal and other simple decision-making performance has resulted in a theoretical gap between the areas of interval timing and perceptual decision-making.

We show that an overarching decision-theoretic framework that encompasses existing, non-temporal decision models can nevertheless

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account both for interval timing itself and for time-based decision-making. We previously showed (Simen, Balcı, deSouza, Cohen, & Holmes, 2011a) that a noisy evidence-accumulation process (specifically, a drift–diffusion process) can account for well-known patterns typically observed in simple, timed responding, such as unbiased estimation and timescale invariance of response time distributions, as well as for new, predicted patterns, such as one-trial learning of intervals, and inverse Gaussian response time distributions with skewness equal to three times their coefficient of variation (standard deviation divided by the mean).

We now go beyond simple, timed responding and demonstrate that a drift–diffusion-based account can explain all the critical features of two-alternative forced-choice (2AFC) tasks in a temporal context. Our results favor a unified theoretical view of timing and both temporal and non-temporal decision-making in terms of drift–diffusion mechanisms, parameterized so as to maximize reward rates earned from repeated decisions. We tested the predictions of this unified theory using one of the most common tasks in the psychophysical study of interval timing: the temporal bisection task.

1.1. Temporal bisection task

In this task, participants are initially trained to discriminate a pair of reference durations, signaled by a stimulus such as a tone or light, as either *short* or *long*. Following this pre-training, participants are presented with a random sequence of short or long reference-duration stimuli and intermediate duration stimuli. Participants are asked to categorize these as *short* or *long* based on their similarity to the two reference durations. Correct categorizations of the reference durations are rewarded; categorizations of intermediate durations and incorrect reference categorizations are not rewarded. The observed proportion of *long* choices as a function of stimulus duration defines a psychometric function of time that is typically sigmoidal. The stimulus duration at which a participant exhibits equal proportions of *short* and *long* choices is known as the point of subjective equality (PSE). The steepness of the psychometric function is an index of the participant's level of timing uncertainty.

In this study, we evaluated a process model of decision-making in temporal bisection as a two-stage drift–diffusion process. Before outlining the basics of this model of temporal bisection, we will briefly describe a single-stage drift–diffusion model of two-alternative forced choice as it is typically applied in a non-temporal context. The model we subsequently propose relies heavily on the same drift–diffusion dynamics.

1.2. The drift–diffusion model

Diffusion models have been successfully applied to two-alternative forced choice in several cognitive domains that include but are not limited to memory (e.g., Ratcliff, 1978. 1988), and perceptual (e.g., Starns & Ratcliff, 2010) and economical decision making (e.g., Krajbich, Lu, Camerer, & Rangel, 2012). The drift–diffusion model (DDM) assumes that sensory information is noisy. The model's decision variable equals the difference between the evidence supporting the two hypotheses, integrated over time. As a result, the variable carries out a random walk, just as a stock price varies over time (see Fig. 1A). When it crosses one of two absorbing boundaries, or decision thresholds, the corresponding decision is made. The first threshold-crossing time is identified as the decision time (DT). An additional non-decision latency (T_{er}) is used to capture sensory encoding (e) and motor response (r) delays. The overall response time is therefore RT = DT + T_{er}.

In its simplest form, the DDM is defined by the starting point (z) and threshold (a) parameters, and an equation governing the decision maker's state of preference x for one or the other choice option. This state of preference changes over time until eventually it descends below 0 or rises above a. Each event corresponds to making one of the
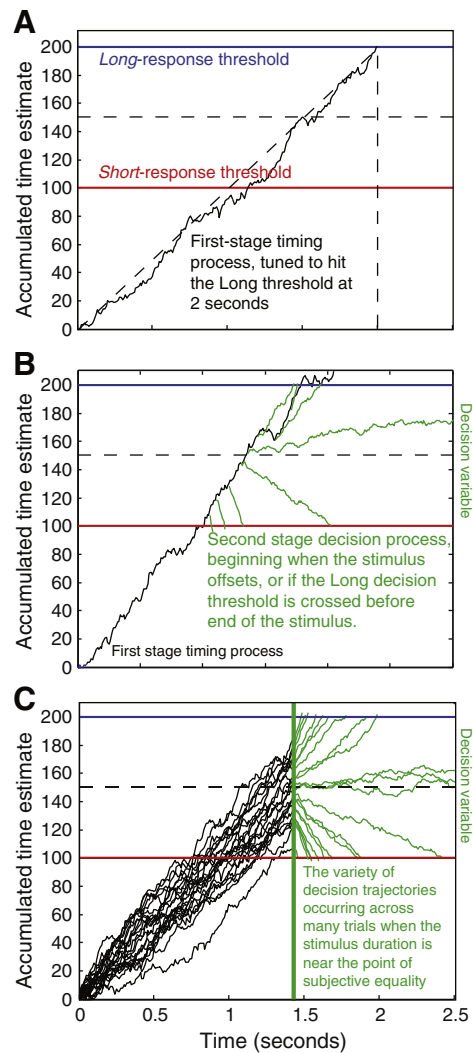
Fig. 1. A) A drift–diffusion process (black) used to track time up to the duration of the long reference duration (when the process intersects the blue threshold). B) A decision process begins when a stimulus duration ends. Seven decision processes (green) are shown for seven different stimulus durations. The starting point of the decision process equals the location of the timer process at the end of the stimulus. Note that the decision process starts at different locations depending on where the first process is at the end of the stimulus duration. Drift is toward the long threshold if the timer location exceeds the level of subjective equality (black dashed line). C) An ensemble of trials with a stimulus duration of 1.4 s. Since this time is near the point of subjective equality, the decision process has a distribution of starting points (centered on black dashed line) and drift values (with mean 0). Note that Fig. 1A only shows the first stage timer process, Fig. 1B shows the trajectory of the second-stage decision process for different stimulus durations (for a given first stage timer trajectory), and Fig. 1C shows the first and second stage trajectories for a given stimulus duration.

two possible choices, so we refer to the 0 and a levels respectively as the *lower* and *upper response thresholds*. Technical details of the diffusion model, including the generalized form of it (Ratcliff, 1978) typically seen in the literature, are described further in Supplementary Online Material.

In our case, the two hypotheses are *short* vs. *long* (without loss of generality, we can assume that upper threshold crossings produce a *long* response, while lower threshold crossings produce *short* responses). Another key parameter is "v", which represents the average rate of increase in X over time. Biases toward either type of response can be built in by moving the starting point z closer to either a or 0; perfectly unbiased responding occurs when $z = a / 2$. We can simulate the DDM's evolution over time with the following simple difference equation, in

which a fixed quantity is added to X at each time step of duration Δt, and also a Gaussian random variable is added:

$$X(t + \Delta t) = X(t) + v \cdot \Delta t + c \cdot Z(t) \cdot \sqrt{\Delta t}. \tag{1}$$

This reduced form of the DDM assumes no trial-to-trial variability in its core parameters: drift rate v, nondecision time T_{er}, and starting point z (equal to $X(0)$). The well-known Ratcliff diffusion model, in contrast, posits variability in v, T_{er} and z from trial to trial (Ratcliff & Rouder, 1998; Ratcliff, Van Zandt, & McKoon, 1999; see Ratcliff & McKoon, 2008; Wagenmakers, 2009 for detailed reviews of the DDM). The reduced model of Eq. (1) predicts equal mean response times for correct decisions and errors when $z = a / 2$ — a prediction that is usually violated in empirical data (Luce, 1986). The Ratcliff model's variability parameters give it the flexibility to fit unequal correct and error RTs and generally enable close fits to empirical two-choice RT data. However, the reduced DDM offers advantages in terms of analytical tractability (Bogacz, Shea-Brown, Moehlis, Holmes, & Cohen, 2006), and possibly in terms of its ability to force the core parameters to absorb maximal variance in the data (Simen et al., 2009), thereby highlighting changes in their values across task conditions. We fit a model of this reduced type to our bisection data later.

1.3. Two-stage sequential diffusion model of temporal bisection

We assumed a sequential drift–diffusion model for performance of the temporal bisection task where during the first stage an estimate of the stimulus duration is achieved and during the second, this time estimate is classified as either closer to the short or the long reference duration. There is a well-defined dependency between the two stages; a noisy estimate of the stimulus duration made during the first stage determines the drift rate of the second stage decision process. More specifically, in the first stage, a drift–diffusion process times the stimulus duration (Rivest & Bengio, 2011; Simen et al., 2011a); in the second, long/short decisions are made by a drift–diffusion comparison process (e.g., Ratcliff, 1978; 1981; 1985; 1988; 2002; Ratcliff & Rouder, 1998; Ratcliff et al., 1999; Smith, 2000) that functions much like the model just described. The second-stage comparison process's starting point and drift parameters are determined by the difference between an estimated stimulus duration and an estimated bisection point that divides "short" durations from "long". As with process models of non-temporal perceptual decision making, which extend signal detection theory into an account of response times as well as accuracy (see Stone, 1960), this approach enables an account of both response time and accuracy in temporal decision making. Below, we describe these two stages in more detail.

1.3.1. First stage: diffusion model of timing

Firstly, the decision-maker should keep track of time to be able to make accurate decisions about time intervals. The first stage of the model does just that, namely it keeps track of time. The model makes *short–long* decisions on the basis of a noisy time estimate, which is itself obtained by accumulating a noisy clock signal as described in Rivest and Bengio (2011), Simen et al. (2011a), and Simen, Rivest, Ludvig, Balci, and Killeen (2013). In this model, a drift–diffusion process with a single absorbing boundary keeps track of time intervals; time intervals are tracked by accumulating a noisy signal with a constant mean value. This accumulation process is not perfect, however, and is subject to normally distributed random noise with mean 0. The accumulation perturbed by random noise continues until it crosses a single, fixed threshold. The first threshold crossing times constitute the estimation of the corresponding time interval.

One of the core assumptions of the model is that drift and diffusion arise from accumulating Poisson pulses, some excitatory, and some inhibitory (Simen et al., 2011a; Simen et al., 2013). When excitation and inhibition are always in constant proportion to each other, then the net pulse count is approximately a drift–diffusion process with noise co-efficient proportional to the square root of the drift rate. This is a crucial assumption, as it predicts that the first threshold-crossing times of the accumulation trajectories exhibit time-scale invariance: that is, distributions of response times with different means superimpose upon each other when divided by the mean response time. This feature of the model accounts for empirical response time distributions in timing tasks, which usually have this property (e.g., Gibbon, 1977; Malapani & Fairhurst, 2002). We refer to this DDM arising from a balance between excitation and inhibition as a time-adaptive, opponent Poisson DDM (TOPDDM).

Within the TOPDDM framework, different intervals are timed by adjusting the accumulation rate; a higher drift rate is used to time shorter intervals (cf. Killeen & Fetterman, 1988). Specifically, in order to time intervals accurately, the drift rate is set to a value equal to the threshold value divided by the target interval. Simple learning rules of the model (Rivest & Bengio, 2011; Simen et al., 2011a) can adjust the accumulation rates to veridical values after a single experience of a given interval, and behavioral evidence supports this form of maximally rapid learning (Bevins & Ayres, 1995; Davis, Schlesinger, & Sorenson, 1989; Simen et al., 2011a). In the first stage of the model, the accumulation rate is parameterized to time the long interval, which can be learned even in a single trial during training. Following training the target time is assumed to stay the same. Previous studies have also shown that participants aim at a single criterion duration during temporal bisection (Allan, 2002; Balcı & Gallistel, 2006). The TOPDDM thus constitutes a drift–diffusion account of interval timing that estimates time in a psychologically and neurally plausible manner.[1] It serves here as the initial stage of a sequential decision process, in which the decision maker dynamically adapts its response biases during the course of a trial by keeping track of time, prior to the end of the timing stimulus presentation. Note that the adaptive features of TOPDDM were not utilized as core properties of the current model.

1.3.2. Second stage: diffusion model of time discrimination

In temporal bisection, a standard two-alternative decision process can be applied to the output of the timer (a TOPDDM) described above. A two-boundary DDM of 2AFC decisions – which maximizes accurate-responses-per-second in some task conditions (Bogacz et al., 2006) – is approximately optimal in the task we explore here.

We assume that during the second stage, when a stimulus duration has elapsed, an instantaneous comparison process occurs: The current stimulus-duration (represented by how high the first-stage timer has risen) is compared by simple subtraction to an estimated bisection duration (the PSE, represented by the midpoint between the two, second-stage decision thresholds). If the difference between the current duration estimate and the bisection duration exceeds a threshold level, then a decision can be made at that time: the model chooses *long* if the difference is sufficiently positive, and chooses *short* if it is sufficiently negative. Without noise, these choices could be made with perfect accuracy at every moment in time except exactly at the PSE.

If noise in the brain interferes with this comparison process, then better decision accuracy can be obtained by repeatedly sampling and accumulating the evidence provided by the first-stage timer representation (mechanistically, this representation is implemented by observing the location of the timer process at the end of the stimulus duration). It is reasonable to assume that the second stage decision process begins with some prior belief state (a prior log-odds estimate, or starting point) set by where the first stage timer process "stops" at the end of

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[1] Simen, Balcı, deSouza, Cohen, and Holmes (2011b) further showed that sequential drift–diffusion processes with different intrinsic time constants can also time long intervals in a neurally plausible manner.

a stimulus duration. Within this framework, the starting point of the decision process should begin closer to the *short* threshold and move closer to the *long* threshold as stimulus durations increase. This prediction derives from the optimal starting-point shift for two-alternative tasks in which the prior probabilities of each type of stimulus are unequal; in such cases, the optimal shift in the starting point away from the midpoint between the decision thresholds is proportional to the prior log-likelihood ratio of the two stimuli (Edwards, 1965). In the case of temporal bisection, this log-likelihood ratio of *long* vs. *short* should grow with elapsing time while the stimulus is present, increasing from a preference for *short* to a preference for *long*. Similarly, the first-stage diffusion process that determines the starting point of the second stage increases with time, until the stimulus ends.

At this point in the decision process, signal detection theory could be used to model choice probabilities based on the final state of the first-stage timing process; however, no predictions could then be made about decision durations. It is reasonable to assume that the evidence-weighing process occurs in a context of background noise in the brain; this is the same assumption made in typical applications of the DDM to 2AFC decisions about briefly presented stimuli, such as masked letters in letter discrimination tasks (e.g., Ratcliff, Thapar, & McKoon, 2006; Thapar, Ratcliff, & McKoon, 2003). In these tasks, evidence accumulation occurs after the stimulus is no longer present but is merely residing in memory.

In the proposed model of temporal bisection, the drift will be highly correlated with the difference between the starting point and the midpoint between thresholds. We can therefore assume the following relationship between drift and starting point:

$$v = K(z - a/2). \tag{2}$$

We leave the value K of the linear function relating drift to starting point as a free parameter to be fit to data.

Fig. 1 illustrates an implementation of this two-stage diffusion process that carries out the functions of both time estimation and comparison. The first stage, a time-tracking process (Rivest & Bengio, 2011; Simen et al., 2011a), takes place over the course of the timing stimulus (prior to its offset; see black trace in Fig. 1). The second stage, a two-choice decision process with parameters determined by the final state of the timer process (green traces in Fig. 1), usually occurs after the end of the stimulus duration. A temporal distance metric defined between a given stimulus duration and the reference durations (or a criterion between the reference durations; cf. Allan, 2002; Balcı & Gallistel, 2006) can be treated as the evidence to be accumulated. During the second stage, this evidence builds towards one of two thresholds, corresponding to *short* and *long*, at the end of the stimulus duration. The categorization then depends on the threshold that is hit first. During the timing stimulus, the temporal distance metric being estimated by the observer continuously changes with time; in contrast, after the termination of the timing stimulus, the signal (the temporal distance) is assumed to be static, although continuously perturbed by white noise.

1.4. Overview of modeling

The proposed model constitutes the first mechanistic account of temporal bisection performance that includes both choice proportions and response times. Importantly, the model provides an overarching diffusion model of interval timing and temporal decision making in the context of a single task extending the scope of our earlier modeling work to the temporal decision making domain. In this work, we evaluated the proposed sequential drift–diffusion model in the light of temporal bisection data (choice proportions and associated response times) that were collected from human participants. In doing so, we followed two strategies by conducting two independent sets of model fits. In the first set of analyses, we fit the full sequential model (Fig. 1)

just described to choice proportions and the corresponding response times. Fig. 2 shows the response time densities predicted by the proposed model. The Results section of the main text presents the results gathered from the full model fits. We then conducted a second set of analyses in which we only fit the second stage process that represents a more conventional analysis for 2AFC decision-making. Details regarding this second set of analyses are presented in the Supplemental Online Material (SOM).
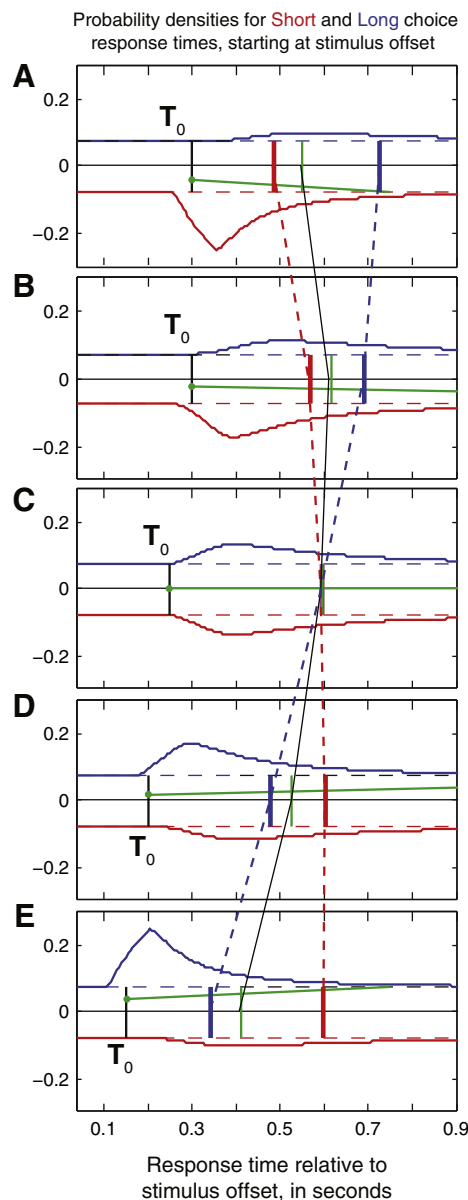
Fig. 2. A–E) Response time densities (long-response: blue, short-response: red) plotted over a deterministic representation of the decision process. Results are shown for a stimulus duration that is subjectively very short (A), short (B), bisecting (C), long (D) and very long (E). Densities are vertically compressed so that the model-parameter illustration between blue and red densities is visible (green line: the average decision process trajectory, with slope equal to the drift parameter; green dot: starting point; black vertical: mean nondecision time T_{er} relative to stimulus offset; red vertical: average short-response RT connected with red dashed lines; blue vertical: average long-response RT connected with blue dashed lines; green vertical: overall average RT connected with green solid lines). For these illustrative simulations, representative values taken from model fits were used. T_{er} decreases with stimulus duration (see text for explanation). Thresholds, however, are held constant for greater visual simplicity.

1.5. Predictions

The model, coupled with the assumption that parameters are tuned to maximize the rate of reward for correct responses per unit time, makes testable predictions regarding the relation of the stimulus duration in a bisection trial to the parameter estimates obtained by model-fitting:

1) The drift rate v should increase monotonically as a function of stimulus duration. (Under the assumption that the upper threshold corresponds to *long* and the lower to *short*, as in Fig. 1, v should start out negative and then increase above zero as stimulus duration increases above the point of subjective equality.)
2) The starting point z should begin closer to the *short* threshold and move closer to the *long* threshold as time elapses; as noted previously, the ideal shift would be proportional to the prior log-likelihood ratio of the long vs. short outcomes (Edwards, 1965).
3) Non-decision time T_{er} should be shorter for the *long* choices, and this difference should be more pronounced with longer stimulus durations. At the beginning of a timing stimulus presentation, the current stimulus-duration estimate is so short that a decision-maker could theoretically categorize it as *short* even before the stimulus duration ended. The obvious problem with this approach is that committing to a *short* response could turn out to be wrong if the timing stimulus persists, and commitment to a choice could be difficult to undo. Less problematically, if the stimulus duration has approached the duration of the long reference duration, then a *long* decision can be made with confidence. (If the *long* threshold is high enough, this sort of fast-guessing will rarely cause anticipatory responses that precede the end of the stimulus duration.) Commitment to a long decision prior to the offset of the stimulus duration offers advantages in regard to non-decision components of the response time; it can reduce the response times by beginning the execution of a motor action before the end of the stimulus duration. These response time gains can be accompanied by motor preparedness that presumably strengthens with elapsed time. Briefly, since participants should only commit to long categorization decisions during the timing stimulus and since the likelihood of the first stage timer hitting the *long* decision-threshold increases with time, non-decision time T_{er} should be shorter for longer stimulus durations. This is implemented with the timing threshold of the first stage diffusion process.
4) Starting point variability should increase as the stimulus durations get longer. This is because more error accrues with longer integration times in the first-stage timer process, resulting in higher variability in the amount of accumulated clock signals by the end of longer time intervals. Note that the state of integration at the end of a time signal constitutes the start point of the second process for the corresponding stimulus duration.

Other predictions of the model are difficult to assess with the current task design and current data-fitting methods. The optimal (reward-maximizing) decision threshold separation, for example, is not easy to compute for a task in which signal discriminability varies from trial to trial (cf. the more tractable task designs analyzed in Bogacz et al., 2006). In any case, some data suggest that thresholds do not change as a function of signal discriminability even with extended practice (Balcı, Simen, et al., 2011). Thus, we do not analyze predictions about threshold height.

2. Methods

2.1. Participants

35 adults (11 males and 24 females) with an average age of 19.71 (std 1.56) years, were recruited via announcements posted online and around the Princeton University campus. The experiment comprised daily sessions (between 1 and 5 sessions per participant). Each session had a fixed duration of 1 h. The experiment was approved by the Institutional Review Panel for Human Subjects of Princeton University, and all participants provided written consent for their participation.

2.2. Stimuli and apparatus

A green square (40 × 40 pixels) that appeared in the middle of the screen with a black background constituted the timing stimulus. The green square disappeared after the programmed stimulus duration had elapsed (reference durations and intermediate durations), which prompted a decision.

The sets of stimulus durations ranged between 710 ms–1420 ms and 1420 ms–2840 ms. Each set of stimulus durations (nine durations per set) was pseudo-randomly assigned to a given session, that is, participants were tested with either 710 ms–1420 ms or 1420 ms–2840 ms pairs in a given session. The intermediate durations were 774, 845, 923, 1001, 1093, 1193, and 1299 ms for the 710 ms–1420 ms pair, and they were 1548, 1689, 1856, 2002, 2187, 2386, 2599 ms for the 1420 ms–2840 ms pair. 32 participants were tested with these two separate sets of stimulus durations (one set per session) and three with a single set of stimulus durations. A majority of the participants were also tested with additional sets of stimulus durations that were determined based on their response time data in an independent 2AFC test. In this paper, we only report the data from the conditions defined by 710–1420 and 1420–2840 ms reference durations and skip other data as these stimulus durations differed substantially between individuals. Results however, did not change with the inclusion of these additional data collected using participant-specific sets of stimulus durations (evaluated only for the analysis presented in the Supplementary Online Material).

The inter-trial intervals were sampled from a truncated exponential with a mean of 500 ms and an upper bound of 1 s. Participants could take a break and restart testing by a key press after every 10th trial (except during the first 20 practice trials). Participants were asked not to count or engage in any activity of sub-division to time intervals. In order to prevent chronometric counting, participants conducted a concurrent two-back numerical working memory task. The display was generated in MATLAB on a Macintosh computer, using the Psychophysics Toolbox extension (Brainard, 1997; Pelli, 1997). Responses were collected with a standard computer keyboard.

2.3. Procedure

2.3.1. Familiarization

Participants were presented with the reference durations of a given session four times in alternating order prior to the practice phase. Text on the screen explicitly informed participants which interval was the short reference and which was the long reference.

2.3.2. Practice

Short and long reference durations were presented in a pseudo-random order. Participants categorized reference durations as *short* or *long* by pressing the "V" and "N" keys, respectively. The practice phase continued for at least 30 trials and until the performance in the last 25 trials was at least 92% correct. For correct categorizations, a brief beep sound was played, and the word "Correct" was presented in green for 750 ms at the top of the screen. For errors, a brief buzzer sound was played, and the word "Incorrect" was presented in red for 750 ms. In the event of wrong key presses (keys other than V or N) or key presses during the timing stimulus, a warning phrase was presented for 1250 ms. Participants could wait as long as they wanted before responding. No score was presented during the practice phase.

2.3.3. Testing

Once participants had completed the practice phase, they were informed that the actual test would begin. They were informed that in

the testing phase, unlike the practice phase, intermediate stimulus durations would be presented intermixed with the reference durations, and that they should categorize each stimulus duration as *short* or *long* based on its similarity to the short and long references. Including the two reference durations, each set contained nine stimulus durations spaced equally on a log scale. These stimulus durations were presented in a pseudo-random order. Participants could earn monetary reward only for the correct categorization of two reference durations used in that session (in 2 out of 9 trials given perfect accuracy). Participants were told that correct categorizations of reference durations would lead to a monetary reward, but any categorization of the intermediate intervals would not. Monetary reward per correct trial was determined such that participants could earn on average at most $20 in a test session if they categorized each reference duration correctly. Monetary reward per correct categorization was identical for each participant. Participants were asked to respond as quickly and as accurately as possible to maximize the monetary reward and were told that the session duration was fixed. The speed of categorizations was as important as accuracy for reward maximization, since session duration was fixed. The total number of trials (and therefore opportunities for reward) depended on the speed of responding. In other words, participants could earn more by increasing the number of trials for a fixed level of categorization accuracy of reference durations. Feedback about the accuracy of categorizations (signaling trial-based monetary reward) was given after each trial that contained a reference duration.

During the testing phase, participants received feedback on every 10th trial about their cumulative performance. After each test block, they were presented with the total monetary reward earned from the timing trials, which was then weighted by their performance on a secondary working-memory memory task (described below). Feedback for correct and incorrect categorizations of the reference durations was the same as in the practice phase. There was no feedback for categorizations of the intermediate intervals. Instead, on these trials an upward arrow appeared on the left side of the screen if the "V" key was pressed and on the right if the "N" key was pressed (without auditory feedback). Fig. 3 depicts the temporal bisection procedure.

Chronometric counting is known to affect the statistical properties of timed responding (e.g., Hinton & Rao, 2004; Rakitin et al., 1998; Rattat & Droit-Volet, 2012). Administration of a secondary task (using numerals) to suppress counting is a common application in the timing literature, although instructing participants not to count is an efficient technique in itself (Rattat & Droit-Volet, 2012). Unlike the practice phase, participants also performed a secondary, two-back working memory task during the testing phase in order to prevent spontaneous chronometric counting. After each categorization, participants were presented with a numeral (sampled randomly from the range 1–9) in the middle of the screen for 750 ms. Participants were told that after some trials,

they would be asked to recognize the numeral they had seen two trials ago. The number of trials between working memory "interrogations" was sampled from a truncated Poisson distribution (lower bound of 3 and upper bound of 30) with a mean of 10. In the interrogation trials, participants were presented with a numeral (with $p = .5$ of matching the target numeral), and they were asked if it matched the numeral they saw two trials back. They were asked to press "V" for *yes* and "N" for *no* responses. Participants received feedback regarding the accuracy of their working-memory decision (for 1000 ms). At the beginning of the experiment, participants were told that their earnings from the timing trials would be weighted by the proportion correct achieved in the working memory task.

2.4. Data analysis

When the stimulus durations were normalized by the short reference duration of the corresponding dataset (stimulus duration/short reference duration), choice proportions gathered from 710 ms–1420 ms to 1420 ms–2840 ms conditions superimposed almost perfectly (see SOM Fig. 8) and thus we pooled data across two different sets (710 ms–1420 ms and 1420 ms–2840 ms) per participant. Response time data and choice data pooled between two sessions per participant were fit. The average number of datum points per participant was around 1055. While pooling the data, the stimulus durations used in each session were divided (normalized) by the short reference of the corresponding session for each participant, and then the data from multiple sessions were pooled into bins defined by the normalized stimulus durations (except for three participants who were tested with a single set of intervals). In other words, the response time data and choice data were aggregated between the corresponding stimulus durations of two different test pairs (e.g., data from the short reference duration of both test pairs were pooled together). This allowed us to express the stimulus durations of each session on a normalized time scale.

To be included in the analysis of reaction times, a participant had to have at least 10 datum points for each categorization type (*short* or *long*) of a given stimulus duration. For a stimulus duration to be included in the repeated measures ANOVA, all participants had to meet this criterion (at least 10 datum points for each categorization) for that stimulus duration. Consequently, for the analysis of *short* response times, the first 6 out of 9 stimulus durations were included and for the analysis of *long* response times, the last 5 out of 9 stimulus durations were included. We applied this rule to ensure that response times for each stimulus duration and participant were estimated from large enough datum points.

We fit the second stage of the sequential diffusion model described earlier in the text to individual participants' data using Matlab's fmincon optimization routine, applied to the chi-squared fit error of the model
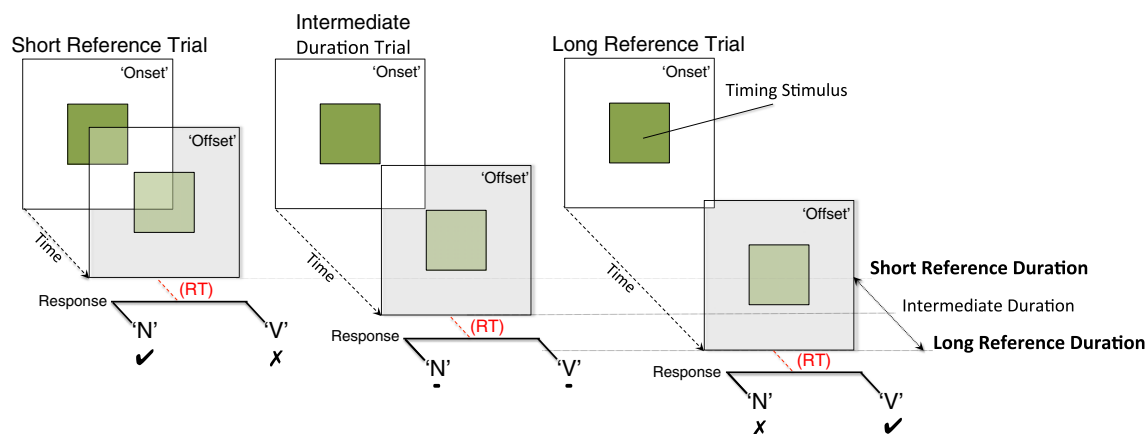

Fig. 3. Depiction of temporal bisection procedure. RT: response time.

versus the data. We used the CDFDif function of Tuerlinckx (2004), borrowed from DMAT (Vandekerckhove & Tuerlinckx, 2008), to compute the cumulative distribution (CDF) of RTs for Ratcliff's diffusion model for a given parameter set, constraining the value of Ratcliff's extra starting point variability and drift variability parameters to 0.0001. We forced the drift parameter to equal a factor K times the difference between the starting point parameter and the midpoint between the two thresholds; K was a free parameter that the fitting algorithm tried to optimize. Finally, we convolved the resulting CDF with a normal distribution of starting points produced by the first stage timer; the noisiness of this timer determined the starting point variability parameter, $\text{Var}(z)$. While fitting the data, we introduced constraints (upper bounds on parameter values) to prevent explosions in parameter estimates (cf. Simen et al., 2009). We confirmed that this fitting software could recover accurate parameter estimates of simulated data sets before applying it to the empirical data.

Results of these fits are presented in the main text, whereas fits of several versions of the Ratcliff diffusion model are presented in the Supplementary Online Material. Ratcliff's diffusion model assumes that starting points and drift are randomly distributed from trial to trial independently of each other. In contrast, in the sequential diffusion model we propose, the starting points and drifts are perfectly correlated. However, aside from this difference, fits of both models ought to produce similar patterns for the core parameters of drift, threshold, starting point, and non-decision latency.

3. Results

3.1. Empirical results

Fig. 4A shows that, as expected, the proportion of long choices is a sigmoidal function of the stimulus durations. Fig. 4B shows the average response times separately for the *short* and *long* choices made while categorizing different stimulus durations (median response time patterns were very similar). This figure shows that the response times of *short* and *long* choices increase with longer and shorter stimulus durations, respectively. In other words, the response times of erroneous categorizations are overall longer than the correct categorizations; here erroneous and correct choices can be defined based on difference between a given stimulus duration and the short and long reference durations: longer than average stimuli are correctly categorized as

long and incorrectly categorized as *short*; shorter than average stimuli should be deemed *short*.

Fig. 4B further shows that the average RT for *long* correct choices is faster than the average RT for *short* correct choices, $t(34) = 11.27$, $p < .001$ (for the combined dataset including three participants tested with a single duration pair; the same results held when these three participants were excluded, $t(31) = 11.25$, $p < .001$). Response times for *short* choices overall, averaged across both corrects and errors, were significantly slower than average response times for *long* choices irrespective of the dataset: for 710–1420 ms $t(33) = 7.68$, $p < .001$; for 1420–2840 ms $t(32) = 9.08$, $p < .001$; for the combined dataset excluding participants tested with a single duration pair, $t(31) = 11.19$, $p < .001$ and for the combined dataset including participants tested with a single duration pair, $t(34) = 11.99$, $p < .001$. *Short* and *long* response times differed significantly across different stimulus durations: $F(5,170) = 15.96$, $p < .001$ (comparing the first 6 stimulus durations) and $F(4,136) = 56.30$, $p < .001$ (comparing the last 5 stimulus durations), respectively. Separate regressions of mean *short* choice RTs and mean *long* choice RTs on the normalized stimulus durations (stimulus duration/short reference duration) both showed that stimulus durations explained a significant proportion of variance in the mean RTs: $F(1,7) = 348.44$, $p < .001$ ($R^2 = .98$) and $F(1,7) = 50.16$, $p < .001$ ($R^2 = .88$) respectively. The same results held when the analyses were conducted on the median response times; $R^2 = .92$ and $R^2 = .97$, respectively (both $ps < .001$).

To show how well the model accounts for patterns in the data without dependence on particular fitting procedures, Fig. 5 shows the pooled RT distributions separately for different stimulus durations and short/long categorizations; compare to Fig. 2.

3.2. Model results

Fig. 6 shows the average sequential DDM parameter estimates from the model fits across all participants. The quality of the fits is depicted in SOM Fig. 9–17, where quantiles of the correct and error RT distributions for the fitted model are plotted against the corresponding quantiles of the empirical data. Visual inspection of these figures suggests good fits for stimulus durations that are closer to the PSE whereas the quality of fits decreased as the stimulus durations approached the reference durations. We note that this model was more constrained than Ratcliff's diffusion model, both structurally, and in numbers of free parameters: drift
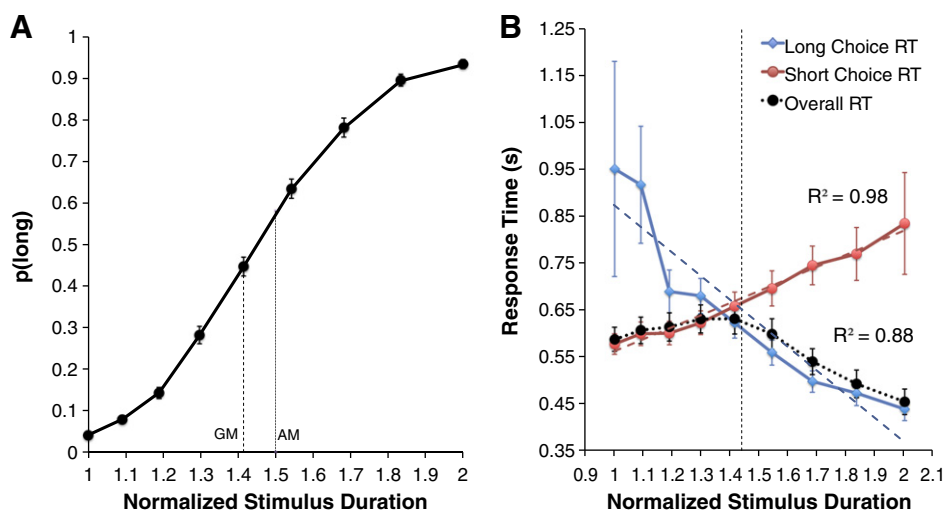

Fig. 4. A) Mean proportion of long choices as a function of the normalized stimulus duration (stimulus duration/short reference duration), with 1 corresponding to the short and 2 corresponding to the long reference duration. Values between 1 and 2 represent the normalized intermediate durations. Error bars denote the SEM. AM: arithmetic mean (dotted vertical line) GM: geometric mean (dashed vertical line). B) Mean *short* (red) and *long* (blue) response times as a function of the normalized stimulus duration. Dashed lines are the best fitting linear regression lines to the data. The dotted curve shows the average overall response times.
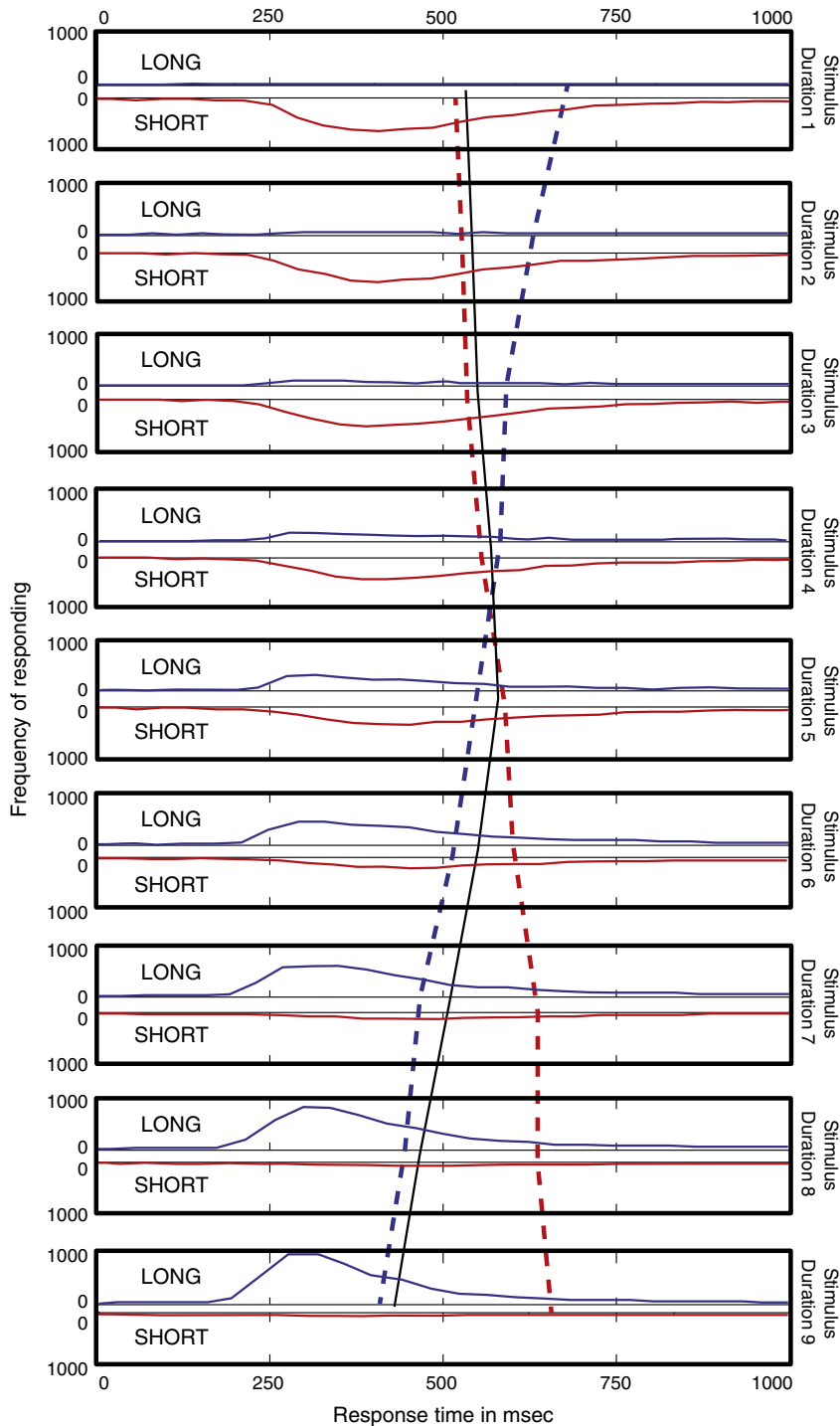
Fig. 5. *Short*-choice (red) and *long*-choice (blue) empirical RT distributions, binned into 40 ms time bins and plotted separately for each stimulus duration, from shortest (1) to longest (9). Red dashed curve denotes average *Short* RT; blue dashed curve denotes average *Long* RT. Black solid curve denotes overall average RT.

and starting point were assumed to be perfectly correlated, and no drift variability was allowed. We did allow variability in the non-decision latency T_{er}. This fitting approach allowed us to estimate starting point noise appropriately for the model; however, we corroborated the main findings for other parameters using DMAT to fit Ratcliff's model with various levels of model complexity (details are in the Supplementary Online Material).

As predicted by the sequential DDM, drift rate increased nearly linearly with increasing stimulus durations (Fig. 6A). Statistical comparison

of estimated drift rates between different stimulus durations revealed a significant overall difference, $F(8,272) = 14.76$, $p < .001$. Visual inspection of Fig. 6B (left axis) suggests that starting point got closer to the long threshold as a function of increasing stimulus duration, supporting the second prediction of the sequential DDM. This observation was corroborated by statistical comparisons, $F(8,272) = 23.83$, $p < .001$. Same results held when starting points were normalized by the threshold values (nominal starting point − decision boundary/2), $F(8,272) = 53.19$, $p < .001$ (Fig. 6B, right axis). The third prediction of the model
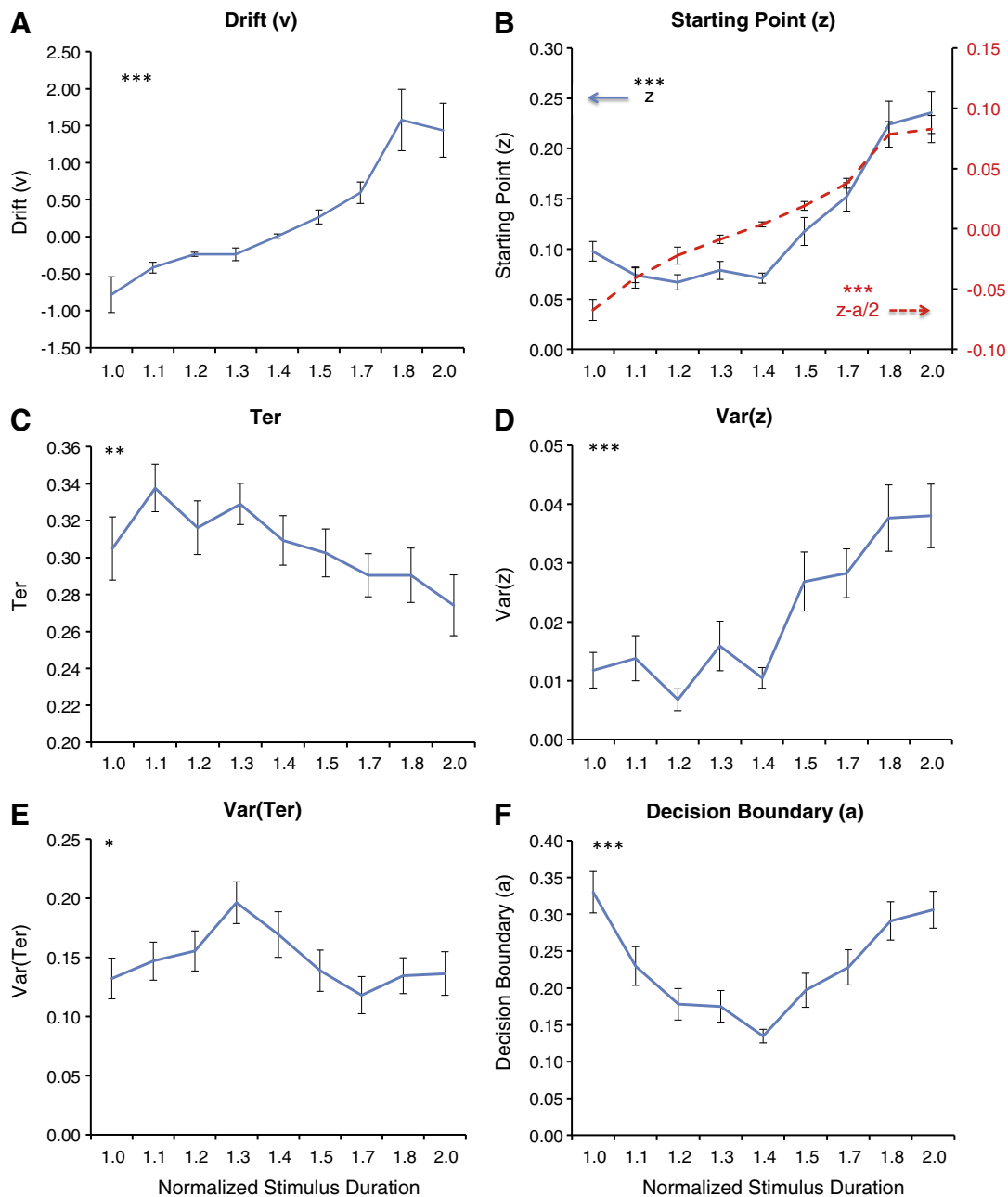
Fig. 6. Second stage diffusion process related parameter estimates. One of the lines in panel B represent the nominal starting points (solid blue lines) whereas the other lines in the same panel represent transformed starting point (starting point-decision boundary/2; dashed red lines). Stars indicate significant (*$p < .05$, **$p < .01$, ***$p < .001$) overall differences between different stimulus durations based on comparisons using repeated measures ANOVA.

was that non-decision time delay would decrease as stimulus durations grew longer due to the interaction between decision commitment and response preparation. Fig. 6C supports such a relation, which was further supported by statistical comparison. There was a significant overall difference of non-decision time between different intervals, $F(8,272) = 3.05$, $p < .01$. The same result held when non-decision times were compared between the last six test durations (over which monotonic decrease is observed), $F(5,170) = 3.50$, $p < .01$.

The final prediction of the model related to starting point variability. Since the starting point was assumed to depend on the state of the first diffusion process at the end of a stimulus duration, the starting point variability of the second stage was predicted to vary more with

increasing duration. Fig. 6D supports this predicted relation (starting point variability $F(8,272) = 8.51$, $p < .001$). One way ANOVA comparisons also revealed significant overall differences between stimulus durations for non-decision time variability $F(8,272) = 2.35$, $p < .05$ and decision boundary, $F(8,272) = 8.48$, $p < .001$. The same results held when three participants tested with a single duration set were excluded from the analyses.

In addition to individual fits, we fit the sequential DDM also to the data pooled between participants per stimulus duration (excluding any response time longer than 2 s). The choice proportion and RT predictions of the sequential DDM are shown along with the empirical data in Fig. 7A. Visual inspection of this figure suggests a close

A

B
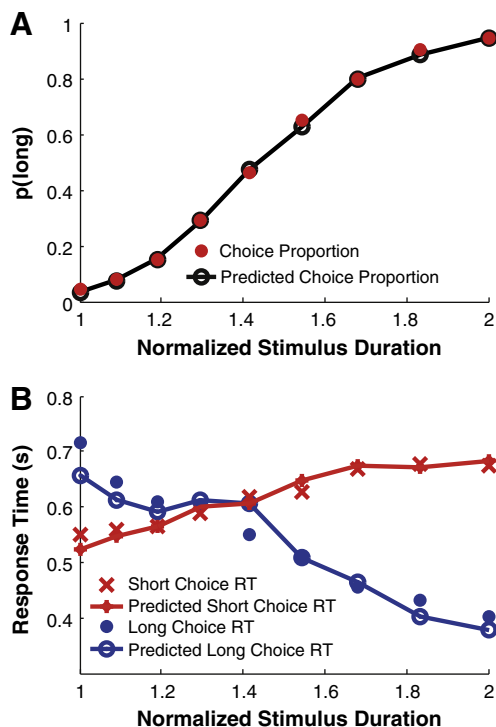
Fig. 7. A) Empirical choice functions computed from data pooled across stimulus duration and participants (red filled circles) and the predicted choice functions (black solid curve with open circles). B) Average short and long choice response times (symbols) estimated from data pooled across stimulus durations and participants. Predicted short and long choice response times (solid lines). Predictions were based on parameter values estimated from fitting the pooled data (after subtracting ~40 ms from the T_{er} estimates). Each simulation was run for 50,000 trials per stimulus duration.

correspondence between empirical and simulation data for both choice proportions and for the RTs (we subtracted ~40 ms from the T_{er} estimates of the model fits to improve the fit quality).

Note that parameter estimates from these fits to pooled data were suspect, with estimates of the K parameter inflated to the constrained fitting procedure's upper bound of 40 for the shortest five stimulus durations. This could account for the 40 ms discrepancy. These pooled-data fits did not reveal the same clear patterns for diffusion model parameters shown in Fig. 6 with fits to individual data. They did, however, fit the choice proportion and average RTs – but not the full RT distributions – better than the average of the predictions based on fits to individuals. Those individual-based predictions were substantially slower than the average RT for "error" responses at the shortest and longest stimulus durations, namely, *long* responses for short stimuli, and *short* responses for long stimuli, probably due to a small number of errors observed for the reference durations. Fits of several different versions of Ratcliff's diffusion model in the Supplemental Online Material corroborate the patterns of parameter estimates obtained from our individual fits here, for all parameters other than our model's starting point variability parameter, which cannot be estimated by fitting Ratcliff's model.

4. Discussion

There has long been a disconnect between the interval timing and decision-making literatures. This disconnect stems from the distinct analytical approaches typically used to examine performance in these two domains, as well as, arguably, from a natural distinction between the types of stimuli used in each. In typical perceptual decision making research, the participant is presented with some kind of signal indicating

which response is correct or likely to be rewarded. The modality, quality, intensity, and duration of that signal are generally strictly controlled by the researcher, along with the equivalent properties of the possible behavioral responses. Almost two hundred years of research in this area has generated psychophysical laws and compelling models of the processes involved, all of which act in a rapid, feed forward manner to transduce the physical energy in a stimulus into a behavioral response. In contrast, the defining feature of the stimuli used in temporal decision making is essentially beyond the researcher's control. The sensory events that indicate the time interval to be remembered can be manipulated, but time itself simply passes by.

If passing time is the signal, though, then in what noise is it embedded? After all, timing abilities, like all other cognitive capacities, are limited in precision. Furthermore, timing-task stimuli frequently cannot be conceived as ongoing sources of physical energy that can be operated on by a fast, feed forward sensory system. Instead, an evolving, internal time estimate must be capable of operating in "open-loop" mode, without any continuous feedback from the environment. How this is accomplished has been a central theoretical question that has occupied timing researchers for decades.

Attempts to answer this question have generated a variety of models of time estimation. The comparison processes used in these models to make decisions about time estimates have usually been considered relatively less theoretically important than the time tracking process itself. Thus, if choice proportion data sufficed to answer some question about how time was tracked in a temporal decision making task, then a detailed investigation of decision times that might characterize the dynamics of the comparison mechanism was of distinctly secondary importance.

We and others have recently shown that borrowing the drift–diffusion concept normally applied to non-temporal decision making allows an exceedingly simple account of the internal time estimation process itself (Rivest & Bengio, 2011; Simen et al., 2011a; Simen et al., 2013). That this account had not previously been proposed suggests once again some amount of disconnect between the perceptual decision making and timing literatures.

What we have now shown is that in the kinds of temporal decision tasks not usually studied in perceptual decision making research, the decision process may be exactly the same as in non-temporal tasks. Furthermore, it is possible to give an account of the patterns of parameter change that occur as a function of temporal signal discriminability. These patterns are consistent with principles of approximate reward maximization, which have similarly been shown to govern behavior in non-temporal decision tasks (Balcı, Simen, et al., 2011; Bogacz, Hu, Holmes, & Cohen, 2010; Simen et al., 2009; Starns & Ratcliff, 2012). Most importantly, we have shown that a simple account of both the time estimation and decision components of a classic temporal decision task can be accomplished by a single type of neurally plausible process: a random walk, or diffusion process.

4.1. Temporal bisection predictions of the DDM

Specifically, we modeled temporal decision-making in the temporal bisection task using the sequential diffusion model. As expected, fitted drift rates in all these models increased monotonically as a function of stimulus duration within a trial. Consistent with the shape of the psychometric function, these patterns suggest that participants made *short–long* decisions by carrying out a drift–diffusion process at the end of the stimulus duration, with drift and starting point determined by the temporal distance of the stimulus duration from an estimated bisection point, and with noise caused exclusively by the brain's internal processing. Mechanistically, this temporal distance could simply be determined by where the first timer stops when the stimulus duration is over.

In addition to the drift rates, nondecision times reliably decreased with stimulus duration (at least over the last six test durations). This

was expected, because participants were free to commit to *long* decisions prior to the end of any stimulus duration. Under the assumptions of the DDM, such pre-commitment would allow participants to reduce response times by beginning the execution of a motor action before the end of the stimulus duration. As measured from the timing stimulus offset, this reduction would appear as a leftward shift of the response time distribution. For unusually long intervals, such response time reductions would only negligibly reduce accuracy, thereby driving up reward rates. Furthermore, once a *long* decision is made, the continuation of elapsing time should only continue to strengthen that commitment. In contrast, it is rarely beneficial to pre-commit to *short* decisions in this task. Stimulus durations were never shorter than the *short* reference duration, and at any time prior to the end of a stimulus duration, the continued elapsing of time could eventually favor a *long* decision even when it currently favors a *short* one. Simulations of the sequential temporal bisection model further showed that these response time gains might be partially due to overall motor preparedness that strengthens with time. This dynamic would clearly primarily favor long categorizations due to the task structure, but it would also speed up even those few *short* responses that occur for relatively long stimulus durations (see SOM Fig. 18).

Under the framework of a two-stage, sequential decision-making process (as described in the Introduction), it is normative to change the starting point of the second comparison process as a function of stimulus duration. Edwards (1965) analyzed optimal DDM parameterizations in two-choice perceptual decisions with unequal prior probabilities of the two hypotheses. Here, in contrast, those prior probabilities evolve dynamically within each trial as time progresses during the timing stimulus. Our model fits support this prediction; the starting point was observed to move from a point closer to the short decision threshold to a point closer to the long decision threshold as stimulus durations increased.

Bogacz et al. (2006) analyzed conditions under which optimal starting points would exceed the optimal thresholds, producing fast-guess responses. Simen et al. (2009) confirmed that simultaneous manipulations of prior probability and response-to-stimulus interval produce the predicted fast-guess form of non-integrative responding in human visual dot-motion discrimination. Critically, fast-guess responding in Simen et al. (2009) was accompanied by large reductions in the non-decision time analogous to those observed here. One feature of the current data that seems inconsistent with reward maximization, however, is that *short* responses for long intervals are faster than *long* responses for short intervals. This could be explained by a suboptimal pre-commitment to *short* responses prior to the end of long intervals on some trials, or more likely, as the result of a general readiness phenomenon in which the motor system simply speeds up as stimulus duration increases.

Within the framework of the proposed sequential diffusion model, one would also expect the trial-to-trial variability to increase for both starting point and drift-rate based on the functional relation of this parameter with the first-stage timer process. This was expected since the distribution of final positions of the first-stage diffusion timing process becomes more variable as stimulus durations increase. This prediction was also nicely supported by our findings.

There are two opposing predictions regarding threshold modulation given the temporal bisection task structure. On the one hand, since only the correct categorization of reference durations was rewarded, one would expect participants to set their decision-thresholds higher for the reference durations and set them lower for intermediate stimulus durations in order to increase the number of trials and therefore the number of reward opportunities. On the other hand, one might also expect participants to set their thresholds lower for stimulus durations for which the signal-to-noise ratio is very high (i.e., reference durations), higher for stimulus durations for which the signal-to-noise ratio is lower but not zero (i.e., intermediate stimulus durations between the references and the PSE), and very low for stimulus duration equal to

the PSE (see Bogacz et al., 2006). These conflicting strategies might plausibly compete in guiding choice behavior (especially for reference durations) in the temporal bisection task.

4.2. Summary

Our results demonstrate that diffusion models can simultaneously account for response times as well as choice proportions in temporal decision-making data. Earlier work has assumed that some process compares experienced intervals with memory representations of critical interval(s), but has not explicitly modeled comparison as a real-time process entailing response time predictions. To our knowledge, this work is the first to examine the dynamical properties of this comparison process in temporal bisection. Our results suggest a unified account of two-choice perceptual and temporal decision making based on diffusion processes.

Our results do not rule out other variants of the diffusion model that could similarly support a temporal decision process. For example, the first stage of the model could be replaced by a timing system employing two parallel timers, one that times the short reference interval, and one that times the long reference; the final decision could then be made on the basis of a combination of both timers' states. For the second stage, the Ornstein–Uhlenbeck model (Usher & McClelland, 2001) for example, generalizes our second stage drift–diffusion process by allowing for a feedback term in the right hand side of Eq. (1). The linear ballistic accumulator model (Brown & Heathcote, 2008), in contrast, eliminates the diffusive noise from Ratcliff's diffusion model, but retains its trial-to-trial variability parameters. There are also a number of other models within the Ratcliff family itself, which do not have the restrictions we placed on most of its trial-to-trial variability parameters.

All of these models can account for the key features of our data set: slower decision making near the point of subjective equality, faster overall RTs near the long reference duration, and positively skewed distributional RT data. Distinguishing among these models is often difficult, and temporal bisection is no different. Our purpose here has been to demonstrate that key features shared by all of these accumulator models can account for the decision process in temporal bisection — indeed, that they can even account for the basic temporal representation used as input to the decision process. The confirmation of our predictions regarding sequential diffusion model parameters in the temporal bisection task suggests that temporal and non-temporal decision making share a common decision process.

Acknowledgments

Appendix. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.actpsy.2014.03.005.

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